$\vartheta \in \Theta$ $\qquad \Theta = \Theta_0 \cup \Theta_1$

$H_0: \vartheta \in \Theta_0 \leftarrow$ ipotesi nulla $\qquad \Theta_0 \cap \Theta_1 = \emptyset$

$H_1: \vartheta \in \Theta_1 \leftarrow$ ipotesi alternativa $\qquad \underbrace{\overbrace{\quad}^{\Theta_0}\overbrace{\quad}^{\Theta_1}}_{\Theta}$

μ_0 = tasso medio di colesterolo in un individuo sano

n pazienti si somministra il farmaco

$X_1, X_2, \dots X_n$

X_i = tasso di colesterolo nell'individuo i-esimo dopo la somministr.

$\rightarrow X_i \sim N(\mu, \sigma^2)$

$H_0: \mu \le \mu_0$ = il farmaco funziona

$H_1: \mu > \mu_0$ = il farmaco non funziona

$\mu \in \mathbb{R}$ $\qquad \vartheta \in \Theta$, $\vartheta \downarrow \mu$, $\Theta \downarrow \mathbb{R}$: $\mu \in \mathbb{R}$

μ_0

$\mathbb{R} = \underbrace{(-\infty, \mu_0]}_{\Theta_0} \cup \underbrace{(\mu_0, +\infty)}_{\Theta_1}$

$H_0: \mu \in \Theta_0$; $H_1: \mu \in \Theta_1$

$T \rightarrow \boxed{\dfrac{X_1 + \dots + X_n}{n} = \overline{X}}$ $\qquad \overline{X} < \mu_0$ $\qquad \mu_0 = 2$

$\overline{X} = 57$

H_0 è l'ipotesi più sfavorevole

$\rightarrow$ Errore di 1ª specie = respingere H_0 a torto

Errore di 2ª specie = accettare H_0 a torto

L'errore di 1ª specie è considerato più grave

$H_0: \vartheta \in \Theta_0$ $\qquad T(X_1, \dots, X_n) = T$

$H_1: \vartheta \in \Theta_1$ $\qquad \{T \in D\} \rightarrow$ respingo H_0

Regione critica del test = $\{T \in D\}$

$\alpha^* = \sup_{\vartheta \in \Theta_0} P^{\vartheta}(T \in D)$ = massima prob. di commettere un errore di 1ª specie

= taglia del test

$\alpha^* \le \alpha$ $\qquad \alpha = 0.05$

$(X_1, \dots, X_n)$ campione gaussiano

$\mathcal{N}(\mu, \sigma^2)$

→ Test per la media

Test per la varianza

(varianza nota)

Test bilaterale	Test unilaterale	Test unilaterale
$H_0 : \mu = \mu_0$	$H_0 : \mu \leq \mu_0$	$H_0 : \mu \geq \mu_0$
$H_1 : \mu \neq \mu_0$ ①	$H_1 : \mu > \mu_0$ ②	$H_1 : \mu < \mu_0$ ③

$$T = \frac{X_1 + \dots + X_n}{n} = \overline{X}$$

$$\rightarrow \boxed{T = \frac{\overline{X} - \mu_0}{\sigma}\sqrt{n}}$$

$\overline{X} \sim \mu$

→ $H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

$$t = \frac{\mu - \mu_0}{\sigma}\sqrt{n}$$

$\{|T| > k\}$ = regione critica

$$\rightarrow \boxed{\alpha^* \leq \alpha}$$

taglia, livello, 0.05

$$\alpha^* = \sup_{\vartheta \in \Theta_0} P^\vartheta(T \in D)$$

$$P^\vartheta(T \in D) = P^\mu(T \in D) = P^\mu(|T| > k) =$$

$$= P^{\mu}\left(\left|\frac{\overline{X} - \mu_0}{\sigma}\sqrt{n}\right| > k\right)$$

$X_i \sim N(\mu, \sigma^2)$

μ_0

$H_0 : \mu \in \underbrace{\{\mu_0\}}_{\Theta_0}$ $\quad H_1 : \mu \notin \{\mu_0\}$

$$\sup_{\vartheta \in \Theta_0} P^\vartheta(T \in D) = \sup_{\mu \in \{\mu_0\}} P^\mu\left(\left|\frac{\overline{X} - \mu_0}{\sigma}\sqrt{n}\right| > k\right)$$

$$= P^{\mu_0}\left(\left|\underbrace{\frac{\overline{X} - \mu_0}{\sigma}\sqrt{n}}_{\sim N(0,1)}\right| > k\right) = P(\underbrace{|T|}_{\sim N(0,1)} > k)$$

$$\underbrace{\frac{\overline{X} - \mu_0}{\sigma}\sqrt{n}} \qquad \overline{X} = \frac{X_1 + \dots + X_n}{n} \qquad X_i \sim N(\mu_0, \sigma^2)$$

$$P(|T| > k) = P(T > k) + P(T < -k) =$$

$$= 1 - P(T \leq k) + P(T \leq -k) =$$

$\Phi(-t) = 1 - \Phi(t)$

$$= 1 - \Phi(k) + \Phi(-k) =$$

$$\alpha^* = \underbrace{2(1 - \Phi(k))}_{=} \leq \alpha$$

$$2(1 - \Phi(k)) = \alpha \rightarrow 1 - \Phi(k) = \frac{\alpha}{2}$$

$$\Phi(k) = 1 - \alpha/2 \leftarrow$$

$$k = \varphi_{1-\alpha/2}$$

$$\{|T| > k\} = \{|T| > \varphi_{1-\frac{\alpha}{2}}\}$$

$H_0 : \vartheta \in \Theta_0$ ← ipotesi nulla

$H_1 : \vartheta \in \Theta_1$ ← ipotesi alternativa

$$(X_1, \dots, X_n) \longrightarrow T(X_1, \dots, X_n)$$

$D =$ insieme dei valori che fanno pensare che è vera H_1

$\{T \in D\}$ = regione critica $\longrightarrow$ respingo H_0

livello

$$\alpha^* = \sup_{\vartheta \in \Theta_0} P^{\vartheta}(T \in D) \le \alpha \quad (\alpha \text{ piccolo})$$

$\{|T| > k\}$ $\quad D = (-\infty, -k) \cup (k, +\infty)$

$$P^{\vartheta}(T \in D) \longrightarrow \vartheta \in \Theta_0$$

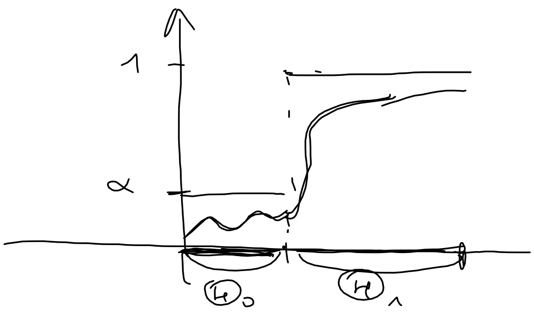
$H_0 : \vartheta \in \Theta_0$

$H_1 : \vartheta \in \Theta_1$

$$\vartheta \in \Theta_1 \longrightarrow \underline{P^{\vartheta}(T \in D)} \longrightarrow \vartheta \in \Theta_1$$

potenza del test

$$\vartheta \longmapsto P^{\vartheta}(T \in D)$$



$$P^{\mu}\left\{ |T| > \varphi_{1-\frac{\alpha}{2}} \right\} = \qquad \mu \neq \mu_0$$

$$= P^{\mu}\left\{ \left| \underbrace{\frac{\bar{X} - \mu_0}{\sigma}\sqrt{n}}_{\not\sim N(0,1)} \right| > \varphi_{1-\alpha/2} \right\} =$$

$$= P^{\mu}\left(\frac{\bar{X} - \mu_0}{\sigma}\sqrt{n} > \varphi_{1-\frac{\alpha}{2}} \right) + P^{\mu}\left(\frac{\bar{X} - \mu_0}{\sigma}\sqrt{n} < -\varphi_{1-\frac{\alpha}{2}} \right)$$

$$= P^{\mu}\left(\underbrace{\frac{\bar{X} - \mu}{\sigma}\sqrt{n}}_{\sim N(0,1)} + \underbrace{\frac{\mu - \mu_0}{\sigma}\sqrt{n}} > \varphi_{1-\frac{\alpha}{2}} \right) + P^{\mu}\left(\underbrace{\frac{\bar{X} - \mu}{\sigma}\sqrt{n}}_{\sim N(0,1)} + \frac{\mu - \mu_0}{\sigma}\sqrt{n} < -\varphi_{1-\frac{\alpha}{2}} \right)$$

$$= P\left(\underset{\sim N(0,1)}{Y} > \underbrace{\varphi_{1-\frac{\alpha}{2}} - \frac{\mu - \mu_0}{\sigma}\sqrt{n}}_{a} \right) + P\left(Y < \underbrace{-\frac{\mu - \mu_0}{\sigma}\sqrt{n} - \varphi_{1-\alpha/2}}_{b} \right)$$

$$P(Y > a) + P(Y < b) =$$

$$= 1 - P(Y \leq a) + P(Y \leq b) = 1 - \Phi(a) + \Phi(b)$$

18 misurazioni della concentrazione di monossido di carbonio $x_1, x_2, x_3, \dots x_{18}$
(mg/m^3)

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_{18}}{18} = \frac{187,73}{18} = 10.43$$

$X_1, X_2, \dots, X_{18}$ campione di legge $N(\mu, \sigma^2)$

$$\sigma^2 = (1,018)^2 (mg/m^3)^2 \qquad \Theta_0 = \{10.00\}$$

$$H_0 : \mu = 10.00 \ mg/m^3 \qquad \alpha = 0.05$$

$$H_1 : \mu \neq 10.00 \ mg/m^3$$

$$T = \frac{\bar{X} - \mu_0}{\sigma} \sqrt{n}$$

$$\longrightarrow \{|T| > \varphi_{1-\frac{\alpha}{2}}\}$$

$$t = \frac{10.43 - 10}{1.018} \sqrt{18} = 1,79$$

$$\varphi_{1-\frac{0.05}{2}} = \varphi_{0.975} = 1.96 \longrightarrow$$ 1.79 non cade nella R.C

dunque non si respinge H_0. ~~Ce~~

Co

quindi $\boxed{\text{al livello } 0.05}$ H_0 è vera

Calc. la potenza del test

per $\mu = 10.50$ mg / m^3

$\vartheta \in \Theta_1 \longrightarrow P^\theta (T \in D)$

$$\Pi(10.50) = 1 - \phi\left(1.96 - \frac{10.50 - 10}{1.018}\sqrt{18}\right) =$$

$$+ \phi\left(-1.96 - \frac{10.50 - 10}{1.018}\sqrt{18}\right) =$$

0.54

$\rightarrow$ $H_0 : p \leq 0.05$ $\qquad \underline{\underline{\alpha = 0.05}}$

$H_1 : p > 0.05$

Si controllano 300 microchip, dei quali 25 risultano difettosi. Cosa si conclude?

$(X_1, \dots X_n)$ campione $\qquad X_i = \begin{cases} 1 & \text{se l'i-esimo pezzo è difettoso} \\ 0 & \text{se no} \end{cases}$

$X_i \sim B(1, p)$

$$T = \frac{\bar{X} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \qquad \{T \geq \varphi_{1-\alpha}\} \leftarrow$$

$$\frac{X_1 + \dots + X_n}{n} = \frac{25}{300} = \bar{x} = \frac{1}{12}$$

$$t = \frac{\frac{1}{12} - 0.05}{\sqrt{\frac{0.05 \cdot 0.95}{300}}} = \underline{2.649} \uparrow \qquad \varphi_{0.95} = \underline{1.64}$$

$H_0 : p \geq 0.05$

$H_1 : p < 0.05$ $\qquad \{T \leq \varphi_\alpha\}$

$$-\varphi_{1-\alpha} = \varphi_\alpha = \varphi_{0.05} = -\varphi_{0.95} = -1.64$$

$T \geq \varphi_{1-\alpha}$ $\qquad \begin{cases} \varphi_{1-\alpha} \leq 2,649 \\ \varphi_\alpha \leq 2,649 \end{cases}$

$T \geq \varphi_\alpha$

$$\begin{cases} \Phi(\varphi_{1-\alpha}) \leq \Phi(2.649) \\ \Phi(\varphi_\alpha) \leq \Phi(2.649) \end{cases} \rightarrow \begin{cases} 1-\alpha \leq 0.996 \\ \alpha \leq 0.996 \end{cases}$$

$\alpha \geq 0.004$

$$\Phi(2.649) = 0,996$$

$$0 \simeq 0.004 \leq \alpha \leq \underline{\underline{0.996}} \simeq 1$$

8 once (oz)

$n = 16$ pacchetti $\qquad \sum_{i=1}^{16} x_i = 126{,}28$ (oz)

$\bar{x}$ $\qquad \sum_{i=1}^{16} x_i^2 = 997{,}141$ (oz²)

(1) $\bar{x}$ e s^2 $\longrightarrow$ $S^2 = \frac{\sum_{i=1}^{n-1} (X_i - \bar{X})^2}{n-1}$

varianza campionaria

(2) $H_0: \mu = 8 \qquad H_1: \mu \neq 8 \qquad \alpha = 0.05$

(3) $H_0: \sigma^2 \leq 0.01 \qquad H_1: \sigma^2 > 0.01 \qquad \alpha = 0.05$

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(1) $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{126.28}{16} = 7.89$

$$s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{16} (x_i - \bar{x})^2}{15}$$

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} (x_i^2 - 2\bar{x}x_i + \bar{x}^2) =$$

$$= \sum_{i=1}^{n} x_i^2 - \underbrace{2\bar{x}}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2 =$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\Big(\underbrace{\frac{\sum x_i}{n}}_{=\bar{x}}\Big) n + \bar{x}^2 n =$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}^2 \cdot n + \bar{x}^2 n = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

$$\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} = \frac{\left(\sum x_i^2\right)}{n-1} - \frac{n}{n-1}\bar{x}^2 = \frac{997.141}{15} - \frac{16}{15}\cdot(7{,}89)^2$$

$$s^2 = 0.032$$

$H_0: \mu = 8 \qquad H_1: \mu \neq 8 \qquad \alpha = 0.05$

$\bar{x} = 7{,}89 \qquad s^2 = 0.032 \qquad n = 16$

$\mu_0 = 8$

$$T = \frac{\bar{X} - 8}{S}\sqrt{16} \rightarrow t = \frac{\bar{x} - 8}{s}\sqrt{16} =$$

$$\left\{ |T| > t_{1-\frac{\alpha}{2}}(n-1) \right\} \qquad = \frac{7.89 - 8}{\sqrt{0.032}} \cdot 4 = -2.41$$

$t_{0.975}(15) = 2.13 \qquad |t| = 2.41$

si respinge H_0

$H_0: \sigma^2 \leq 0.01 \qquad H_0: \sigma^2 \leq \sigma_0^2$

$H_1: \sigma^2 > \sigma_0^2$

$\left\{ T > \chi^2_{1-\alpha}(n-1) \right\} \qquad \sigma_0^2 = 0.01$

$$T = \frac{S^2(n-1)}{\sigma_0^2}$$

$$t = \frac{s^2 \cdot 15}{0.01} = \frac{0.032 \cdot 15}{0.01} = 47{,}61$$

$$\chi^2_{0.95}(15) = 24{,}99$$